# The Pennsylvania System of School Assessment 

## Mathematics Item and Scoring Sampler



## 2023-2024 <br> Grade 8

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## INTRODUCTION

## General Introduction

The Pennsylvania Department of Education (PDE) provides districts and schools with tools to assist in delivering focused instructional programs aligned with the Pennsylvania Core Standards (PCS). These tools include Academic Standards, Assessment Anchors and Eligible Content (AAEC) documents, assessment handbooks, and content-based item and scoring samplers. This Item and Scoring Sampler is a useful tool for Pennsylvania educators in preparing local instructional programs by providing samples of test item types and scored student responses. The item sampler is not designed to be used as a pretest, a curriculum, or any other benchmark for operational testing.

This Item and Scoring Sampler is available in Braille format. For more information regarding Braille, call (717) 901-2238.

## Pennsylvania Core Standards (PCS)

This sampler contains examples of test questions designed to assess the Pennsylvania Assessment Anchors and Eligible Content aligned to the PCS. The Mathematics, Reading, and Writing PSSA transitioned to PCS-based operational Mathematics and English Language Arts assessments starting with the spring 2015 PSSA administration.

The PCS-aligned Assessment Anchors and Eligible Content documents are posted on this portal:
> www.education.pa.gov [Hover over "Data and Reporting," select "Assessment and Accountability," and select "PSSA-PA System of School Assessment." Then select "Assessment Anchors/Eligible Content" on the right side of the screen.]

## What Is Included

This sampler contains test questions, or test "items," that have been written to align to the Assessment Anchors that are based on the PCS. The sample test questions model the types of items that may appear on an operational PSSA. Each sample test question has been through a rigorous review process to ensure alignment with the Assessment Anchors prior to being piloted in an embedded field test within a PSSA assessment and then used operationally on a PSSA assessment. Answer keys, scoring guidelines, and any related stimulus material are also included. Additionally, sample student responses are provided with each open-ended (OE) item to demonstrate the range of responses that students provided in response to these items.

## Purpose and Uses

The items in this sampler may be used ${ }^{1}$ as examples for creating assessment items at the classroom level. Classroom teachers may find it beneficial to have students respond to the open-ended item in this sampler. Educators may then use the sampler as a guide to score the responses either independently or together with colleagues within a school or district. This sampler also includes the General Description of Scoring Guidelines for Mathematics Open-Ended Items that students will have access to during a PSSA mathematics administration. The general description of scoring guidelines may be distributed to students for use during local assessments and may also be used by educators when scoring local assessments.

## Item Format and Scoring Guidelines

The multiple-choice (MC) items have four answer choices. Each correct response to an MC item is worth one point.

Each OE item is designed to take approximately ten to fifteen minutes to complete. During the administration of the PSSA, students are given additional time as necessary to complete the test items. Each OE item in mathematics is scored using an item-specific scoring guideline based on a $0-4$-point scale. In this sampler, every item-specific scoring guideline is combined with examples of student responses that represent each score point to form a practical, item-specific scoring guide.

## Item Alignment

All PSSA items are aligned to statements and specifications included in the Assessment Anchors and Eligible Content Aligned to the Pennsylvania Core Standards. The mathematics content, process skills, directives, and action statements included in the PSSA mathematics questions align with the Assessment Anchor Content Standards. The Eligible Content statements represent the limits of the content of the mathematics questions.

[^0]
## Testing Time and Mode of Testing Delivery for the PSSA

The PSSA is delivered in a traditional paper-and-pencil format as well as in an online format. The estimated time to respond to a test question is the same for both methods of test delivery. The following table shows the estimated response time for each item type.

| Mathematics Item Type | MC | OE |
| :---: | :---: | :---: |
| Estimated Response Time <br> (minutes) | 2 | 10 to 15 |

During an official test administration, students are given as much additional time as is necessary to complete the test questions.

## Mathematics Reporting Categories

The Assessment Anchors are organized into four classifications as listed below.

| $\bullet$ | A = Numbers and Operations |
| :--- | :--- |
| $\bullet \quad$ B $=$ Algebraic Concepts | $\bullet \quad$ C = Geometry |

These four classifications are used throughout the grade levels. In addition to these classifications, there are five Reporting Categories for each grade level. The first letter of each Reporting Category represents the classification; the second letter represents the Domain as stated in the Common Core State Standards for Mathematics. Listed below are the Reporting Categories for Grade 8.

- $\mathrm{A}-\mathrm{N}=$ The Number System
- $B-E=$ Expressions and Equations
- $B-F=$ Functions
- $\mathrm{C}-\mathrm{G}=$ Geometry
- D-S = Statistics and Probability

Examples of MC and OE items assessing these categories are included in this sampler.

## Item and Scoring Sampler Format

This sampler includes the test directions and scoring guidelines that appear in the PSSA Mathematics assessments. Each MC item is followed by a table that includes the item alignment, the answer key, the depth of knowledge (DOK) level, the percentage ${ }^{2}$ of students who chose each answer option, and a brief answer-option analysis or rationale. The OE item is followed by a table that includes the item alignment, the DOK level, and the mean student score. Additionally, each of the included item-specific scoring guidelines is combined with sample student responses representing each score point to form a practical item-specific scoring guide. The General Description of Scoring Guidelines for Mathematics Open-Ended Items used to develop the itemspecific scoring guidelines should be used if any additional item-specific scoring guidelines are created for use within local instructional programs. The student responses in this item and scoring sampler are actual student responses; however, the handwriting has been changed to protect the students' identities and to make the item and scoring sampler accessible to as many people as possible.

## Example Multiple-Choice Item Information Table

Item Information

| Alignment | Assigned AAEC |
| :--- | :--- |
| Answer Key | Correct Answer |
| Depth of Knowledge | Assigned DOK |
| $p$-value A | Percentage of students who selected option A |
| $p$-value B | Percentage of students who selected option B |
| $p$-value C | Percentage of students who selected option C |
| $p$-value D | Percentage of students who selected option D |
| Option Annotations | Brief answer-option analysis or rationale |
|  |  |
|  |  |

Example Open-Ended Item Information Table

| Alignment | Assigned <br> AAEC | Depth of <br> Knowledge | Assigned <br> DOK | Mean Score | Average <br> Score |
| :---: | :---: | :---: | :---: | :--- | :--- |

[^1]
## General Description of Scoring Guidelines for Mathematics Open-Ended Items

4- The response demonstrates a thorough understanding of the mathematical concepts and procedures required by the task.

The response provides correct answer(s) with clear and complete mathematical procedures shown and a correct explanation, as required by the task. The response may contain a minor "blemish" or omission in work or explanation that does not detract from demonstrating a thorough understanding.

3- The response demonstrates a general understanding of the mathematical concepts and procedures required by the task.

The response and explanation (as required by the task) are mostly complete and correct. The response may have minor errors or omissions that do not detract from demonstrating a general understanding.

2- The response demonstrates a partial understanding of the mathematical concepts and procedures required by the task.

The response is somewhat correct with partial understanding of the required mathematical concepts and/or procedures demonstrated and/or explained. The response may contain some work that is incomplete or unclear.

1- The response demonstrates a minimal understanding of the mathematical concepts and procedures required by the task.

0 - The response has no correct answer and insufficient evidence to demonstrate any understanding of the mathematical concepts and procedures required by the task for that grade level.

The response may show only information copied from the question.
Special Categories within zero reported separately:
BLK (blank).............Is blank, is entirely erased, or gives a written refusal to respond
OT. $\qquad$ Is off-task

LOE. $\qquad$ Is in a language other than English

IL $\qquad$ Is illegible

## Grade 8 Formula Sheet

Formulas that you may need on this test are found below.
You may refer back to this page at any time during the mathematics test.

## Exponential Properties

$$
\begin{aligned}
a^{m} \cdot a^{n} & =a^{m+n} \\
\left(a^{m}\right)^{n} & =a^{m \cdot n} \\
\frac{a^{m}}{a^{n}} & =a^{m-n} \\
a^{-1} & =\frac{1}{a}
\end{aligned}
$$

## Algebraic Equations

$$
\text { Slope: } \quad m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}
$$

Slope-Intercept Form: $\quad y=m x+b$
Pythagorean Theorem


$$
a^{2}+b^{2}=c^{2}
$$

Cone


$$
V=\frac{1}{3} \pi r^{2} h
$$

## Cylinder



$$
V=\pi r^{2} h
$$

Sphere


$$
V=\frac{4}{3} \pi r^{3}
$$

## MATHEMATICS TEST DIRECTIONS

On the following pages are the mathematics questions.

- You may not use a calculator for questions 1-2. You may use a calculator for all other questions on this test.


## Directions for Multiple-Choice Questions

Some questions will ask you to select an answer from among four choices.
For the multiple-choice questions:

- First solve the problem on scratch paper.
- Choose the correct answer and record your choice in the answer booklet.
- If none of the choices matches your answer, go back and check your work for possible errors.
- Only one of the answers provided is the correct response.


## Directions for Open-Ended Questions

Some questions will require you to write your response.
For the open-ended questions:

- These questions have more than one part. Be sure to read the directions carefully.
- You cannot receive the highest score for an open-ended question without completing all tasks in the question. For example, if the question asks you to show your work or explain your reasoning, be sure to show your work or explain your reasoning in the space provided.
- If the question does not ask you to show your work or explain your reasoning, you may use the space provided, but only those parts of your response that the question specifically asks for will be scored.
- Write your response in the appropriate location within the response box in the answer booklet. Some answers may require graphing, plotting, labeling, drawing, or shading. If you use scratch paper, be sure to transfer your final response and any needed work or reasoning to the answer booklet.


## MULTIPLE-CHOICE ITEMS

1. What is $\sqrt[3]{8}$ ?
A. 2
B. 3
C. 4
D. 5

## Item Information

| Alignment | B-E.1.1.2 |
| :--- | :--- |
| Answer Key | A |
| Depth of Knowledge | 1 |
| $p$-value A | $55 \%$ (correct answer) |
| $p$-value B | $15 \%$ |
| $p$-value C | $18 \%$ |
| $p$-value D | $12 \%$ |
| Option Annotations | A. Correct: recognizes that $8=2 \bullet 2 \bullet 2=2^{3}$ <br>  <br>  <br>  <br>  <br>  <br>  <br>  <br> C. selects the degree of the root <br> D. computes as $8-3$ |

## Question 2 in this sampler is to be solved without the use of a calculator.

2. A national park covers approximately $3 \times 10^{5}$ acres of land. A state park covers approximately $5 \times 10^{3}$ acres of land. The land area of the national park is approximately how many times as many acres as the land area of the state park?
A. 6
B. 17
C. 60
D. 167

Item Information

| Alignment | B-E.1.1.3 |
| :---: | :---: |
| Answer Key | C |
| Depth of Knowledge | 2 |
| $p$-value A | 18\% |
| $p$-value B | 14\% |
| $p$-value C | 53\% (correct answer) |
| $p$-value D | 15\% |
| Option Annotations | A. divides 3 by 5 but then multiplies the quotient (0.6) by 10 rather than by $10^{2}$ <br> B. divides 5 by 3 and then multiplies the quotient $\left(\frac{5}{3}\right)$ by 10 rather than by $10^{2}$ <br> C. Correct: recognizes $3 \times 10^{5}$ as the larger value, divides the coefficients as $\frac{3}{5}=0.6$ and the powers of 10 as $\frac{10^{5}}{10^{3}}=10^{5-3}=10^{2}$, and then solves $0.6 \times 10^{2}=60$ <br> D. divides 5 by 3 and then multiplies the quotient $\left(\frac{5}{3}\right)$ by $10^{2}$ |

## A calculator is permitted for use in solving questions 3-16 in this sampler.

3. Brian claims that the product of a rational number and an irrational number is always irrational. Which counterexample proves that Brian's claim is not true?
A. multiplying 0 by $\frac{2}{3}$
B. multiplying 0 by $\sqrt{2}$
C. multiplying $\frac{1}{2}$ by $\sqrt{2}$
D. multiplying $\sqrt{2}$ by $\sqrt{2}$

Item Information

| Alignment | A-N.1.1.1 |
| :--- | :--- |
| Answer Key | B |
| Depth of Knowledge | 2 |
| $p$-value A | $18 \%$ |
| $p$-value B | $32 \%$ (correct answer) |
| $p$-value C | $28 \%$ |
| $p$-value D | $22 \%$ |
| Option Annotations | A. selects two rational numbers with a rational product <br> B. Correct: recognizes that 0 is a rational number, $\sqrt{2}$ is an irrational <br>  <br>  <br>  <br>  <br> C. selects a rational number and irrational number whose product is <br> D. selects two irrational numbers with a rational product |

4. One number in the list below is a rational number.

$$
\begin{array}{llll}
\frac{\sqrt{2}}{2} & \frac{\sqrt{6}}{2} & \frac{\sqrt{27}}{15} & \frac{\sqrt{36}}{15}
\end{array}
$$

Which number line shows point N located at the value of the rational number?
A.

B.

C.

D.


| Alignment | $\begin{aligned} & \text { A-N.1.1.5 } \\ & \text { A-N.1.1.1 } \end{aligned}$ |
| :---: | :---: |
| Answer Key | A |
| Depth of Knowledge | 2 |
| $p$-value A | 39\% (correct answer) |
| $p$-value B | 34\% |
| $p$-value C | 14\% |
| $p$-value D | 13\% |
| Option Annotations | A. Correct: recognizes that $\frac{\sqrt{36}}{15}$ is equal to $\frac{6}{15}$, which is equivalent to 0.4 <br> B. evaluates $\frac{\sqrt{2}}{2}$ as $\sqrt{\frac{2}{2}}=\sqrt{1}=1$ <br> C. evaluates $\frac{\sqrt{6}}{2}$ as $\frac{6 \div 2}{2}=\frac{3}{2}=1.5$ (i.e., considers the square root symbol to mean "divide by 2 ") <br> D. evaluates $\frac{\sqrt{27}}{15}$ as $\frac{27}{15}=1.8$ (i.e., does not consider the square root symbol) |

5. Solve for $x: 7^{2}=x^{2}-3^{2}$
A. $\sqrt{10}$
B. $\sqrt{20}$
C. $\sqrt{42}$
D. $\sqrt{58}$

Item Information

| Alignment | B-E.1.1 |
| :--- | :--- |
| Answer Key | D |
| Depth of Knowledge | 1 |
| $p$-value A | $18 \%$ |
| $p$-value B | $15 \%$ |
| $p$-value C | $21 \%$ |
| $p$-value D | $46 \%$ (correct answer) |
| Option Annotations | A. calculates $\sqrt{7+3}$ <br>  <br>  <br>  <br>  <br>  <br>  <br> B. uses $7^{2}=14$ and $3^{2}=6$ to calculate $\sqrt{14+6}$ <br> C. Calculates $\sqrt{7} \bullet 3 \bullet 2$ |
| D. Correct: solves as $7^{2}=x^{2}-3^{2} \rightarrow 49=x^{2}-9 \rightarrow 58=x^{2} \rightarrow \sqrt{58}=x$ |  |

6. Kevin writes the population of his city in the form $a \times 10^{5}$. He also writes the average income of the people in his city in the form $b \times 10^{4}$. To find the total income for all the people in his city, he multiplies his two values. Kevin says his product will be of the form $c \times 10^{9}$ when written in scientific notation. Which statement explains whether Kevin is correct?
A. Kevin is correct only if $a b<10$.
B. Kevin is correct only if $a+b<10$.
C. Kevin is not correct because the exponents are subtracted when multiplying powers with the same base, and 5-4=1.
D. Kevin is not correct because the exponents are multiplied when multiplying powers with the same base, and $5 \times 4=20$.

Item Information

| Alignment | B-E.1.1.4 |
| :--- | :--- |
| Answer Key | A |
| Depth of Knowledge | 2 |
| $p$-value A | $34 \%$ (correct answer) |
| $p$-value B | $30 \%$ |
| $p$-value C | $17 \%$ |
| $p$-value D | $19 \%$ |
| Option Annotations | A.Correct: multiplies the coefficients as ab and the powers as <br> $10^{5} \cdot 10^{4}=10^{5}+4=10^{9}$, resulting in the product ab $\times 10^{9}$, and then <br> recognizes that the form $c \times 10^{9}$ is in scientific notation only when $c$ <br> (which represents the product ab) is less than $10[$ Note: since $a$ and <br> b are both at least 1 and less than 10, the product ab would also be <br> at least 1 but less than 100] <br> B. adds the coefficients rather than multiplying the coefficients <br> C. subtracts the exponents |
| D. multiplies the exponents |  |

7. Lily is making cookies. The graph below shows the linear relationship between the number of cups of butter and the number of cups of sugar she uses.

Ingredients


How many cups of sugar does Lily use for 1 cup of butter?
A. $\frac{3}{4}$
B. $1 \frac{1}{2}$
C. $2 \frac{1}{4}$
D. 3

Item Information

| Alignment | B-E.2.1 |
| :--- | :--- |
| Answer Key | C |
| Depth of Knowledge | 2 |
| $p$-value A | $15 \%$ |
| $p$-value B | $18 \%$ |
| $p$-value C | $59 \%$ (correct answer) |
| $p$-value D | $8 \%$ |
| Option Annotations | A. determines the rate of change in cups of sugar for every $\frac{1}{3}$ cup | of butter

B. determines the amount of change in cups of sugar from $\frac{1}{3}$ to 1 cup of butter
C. Correct: either recognizes that $\frac{3}{4}$ cup of sugar is used for every $\frac{1}{3}$ cup of butter used, divides $\frac{3}{4}$ by $\frac{1}{3}$ to determine that $\frac{9}{4}$ cups of sugar would be used for 1 cup of butter, and then rewrites $\frac{9}{4}$ as $2 \frac{1}{4}$ OR starting at the origin, recognizes that the pattern is "up 3, right 2" and extends this pattern from the point at $\left(\frac{2}{3}, 1 \frac{1}{2}\right)$ to arrive at the ordered pair $\left(1,2 \frac{1}{4}\right)$, identifying the $y$-coordinate $\left(2 \frac{1}{4}\right)$ as the number of cups of sugar for 1 cup of butter (the $x$-coordinate)
D. sees that the amount of butter doubles from the first plotted point to the second plotted point, so doubles the $y$-coordinate $\left(1 \frac{1}{2}\right)$ to locate the next point, thinking that the $x$-coordinate for that ordered pair will be 1 (i.e., does not consider that both coordinates should double)
8. Kara uses 4 red lights for every 9 purple lights in an art display. Which graph models the relationship between the number of red lights $(x)$ and the number of purple lights $(y)$ Kara uses?
A.

B.

C.

D.
Kara's Art Display


Item Information

| Alignment | B-E.2.1.1 |
| :--- | :--- |
| Answer Key | D |
| Depth of Knowledge | 2 |
| $p$-value A | $13 \%$ |
| $p$-value B | $7 \%$ |
| $p$-value C | $9 \%$ |
| $p$-value D | $71 \%$ (correct answer) |
| Option Annotations | A. determines the unit rate is $\frac{4}{9}$ (i.e., inverts the rate of change) <br> B. determines the unit rate is 5 since $9-4=5$ <br> C.determines the unit rate is 4 since Kara uses 4 red lights <br>  <br>  <br>  <br>  <br>  <br> D. Correct: either determines the unit rate is $\frac{9}{4}$ (purple lights over red <br> lights) and identifies the graph that has a slope of $\frac{9}{4}$ OR recognizes <br> that the ordered pairs should be (number of red lights, number of <br> purple lights) and identifies the graph with a line that starts at (0, 0) <br> and passes through (4, 9) |

9. The graph below shows a line and triangles $A$ and $B$.


Which statement about the graph is correct?
A. The slope of the line is $\frac{-3}{2}$, and the length of the hypotenuse of triangle $B$ is $\frac{3}{2}$ times the length of the hypotenuse of triangle $A$.
B. The slope of the line is $\frac{-3}{2}$, and the length of the hypotenuse of triangle $B$ is 2 times the length of the hypotenuse of triangle $A$.
C. The slope of the line is $\frac{-2}{3}$, and the length of the hypotenuse of triangle $B$ is $\frac{3}{2}$ times the length of the hypotenuse of triangle $A$.
D. The slope of the line is $\frac{-2}{3}$, and the length of the hypotenuse of triangle $B$ is 2 times the length of the hypotenuse of triangle $A$.

Item Information

| Alignment | B-E.2.1.2 |
| :---: | :---: |
| Answer Key | D |
| Depth of Knowledge | 2 |
| $p$-value A | 12\% |
| $p$-value B | 21\% |
| $p$-value C | 19\% |
| $p$-value D | 48\% (correct answer) |
| Option Annotations | A. either determines the change in $x$-coordinates from right to left rather than from left to right OR calculates the slope using the change in $x$-coordinates divided by the change in $y$-coordinates (i.e., uses "run over rise" rather than "rise over run") and uses this ratio as the ratio between the side lengths of the similar triangles <br> B. recognizes that the side lengths of triangle $B$ are twice the side lengths of triangle $A$, but calculates the slope using the change in $x$-coordinates divided by the change in $y$-coordinates (i.e., uses "run over rise" instead of "rise over run") <br> C. calculates the slope using the change in $y$-coordinates divided by the change in $x$-coordinates (i.e., uses "rise over run"), resulting in $\frac{-2}{3}$, but then uses the ratio between the lengths of the sides within similar triangles as the ratio (i.e., one leg in triangle $B$ is $\frac{3}{2}$ the length of the other leg) <br> D. Correct: calculates the slope of the line using the change in $y$-coordinates divided by the change in $x$-coordinates (i.e., uses "rise over run"), resulting in a slope of $\frac{-2}{3}$, and recognizes that the side lengths of triangle $B$ are twice the lengths of the corresponding sides in triangle $A$ |

10. The graph below shows the relationship between the time, in hours, and the number of books printed, in thousands, at a book manufacturing company.


Based on the graph, which equation describes the relationship between the time, in hours, and the number of books printed, in thousands?
A. $y=\frac{1}{3} x$
B. $y=x+\frac{1}{3}$
C. $y=3 x$
D. $y=x+3$

Item Information

| Alignment | B-E.2.1.3 |
| :---: | :---: |
| Answer Key | C |
| Depth of Knowledge | 1 |
| $p$-value A | 24\% |
| $p$-value B | 14\% |
| $p$-value C | 52\% (correct answer) |
| $p$-value D | 10\% |
| Option Annotations | A. inverts the slope <br> B. inverts the slope and uses the value of the inverted slope as the $y$-intercept <br> C. Correct: recognizes that a line starting at the origin should be in the form $y=m x$ and that the slope $(m)$ is calculated by dividing the change in $y$-coordinates by the change in $x$-coordinates (i.e., "rise over run"), resulting in a slope of $\frac{3}{1}=3$ for this line <br> D. calculates the slope but uses the value of the slope as the $y$-intercept |

11. Solve: $3 x+8=2(x-5)+x$
A. $x=-18$
B. $x=18$
C. no solutions
D. infinitely many solutions

Item Information

| Alignment | B-E.3.1.1 |
| :--- | :--- |
| Answer Key | C |
| Depth of Knowledge | 1 |
| $p$-value A | $14 \%$ |
| $p$-value B | $16 \%$ |
| $p$-value C | $56 \%$ (correct answer) |
| $p$-value D | $14 \%$ |
| Option Annotations | A.distributes the 2 to $(x-5)$ but omits the " $+x "$ and then solves the <br> equation $3 x+8=2 x-10$ <br> distributes the 2 to $(x-5)$ and to the " $+x "$ and then solves the <br> equation $3 x+8=2 x-10+2 x$ |
|  | C.Correct: distributes the 2 to $(x-5)$, resulting in $2 x-10+x ;$ combines <br> like terms to simplify the expression to $3 x-10 ;$ solves the equation <br> $3 x+8=3 x-10$ by subtracting $3 x$ from each side, resulting in <br> $8=-10 ;$ recognizes that $8=-10$ is a false statement, which means <br> that the equation has no solutions <br> solves the equation, resulting in $8=-10$, recognizes that $8=-10$ is <br> a false statement, but interprets the false statement to mean that the <br> equation has infinitely many solutions rather than no solutions |

12. Two functions are represented by the graph and table below.

Function 1


Function 2

| $x$ | $y$ |
| :---: | :---: |
| -3 | 0 |
| -2 | $\frac{2}{3}$ |
| -1 | $\frac{4}{3}$ |
| 0 | 2 |
| 1 | $\frac{8}{3}$ |

Which statement about the rate of change for the two functions is true?
A. The rate for function 1 is greater because $\frac{3}{2}>\frac{2}{3}$.
B. The rate for function 2 is greater because $\frac{3}{2}>\frac{2}{3}$.
C. The rate for function 1 is greater because it has a greater $y$-intercept.
D. The rate for function 2 is greater because it has a greater $x$-intercept.

Item Information

| Alignment | B-F.1.1.2 |
| :--- | :--- |
| Answer Key | A |
| Depth of Knowledge | 2 |
| $p$-value A | $45 \%$ (correct answer) |
| $p$-value B | $18 \%$ |
| $p$-value C | $25 \%$ |
| $p$-value D | $12 \%$ |
| Option Annotations | A. Correct: calculates the rate of change for function 1 by using the |

change in $y$-coordinates divided by the change in $x$-coordinates (i.e., "rise over run"), resulting in $\frac{3}{2}$, calculates the rate of change for function 2 by using the change in $y$-coordinates divided by the change in $x$-coordinates, resulting in $\frac{2-0}{0--3}=\frac{2}{3}$, and then compares the two rates of change
B. calculates the rate of change for both functions by using the change in $x$-coordinates divided by the change in $y$-coordinates rather than the change in $y$-coordinates divided by the change in $x$-coordinates (i.e., uses "run over rise" rather than "rise over run")
C. uses the $y$-intercept of each function as the rate of change, comparing 3 to 2
D. uses the $x$-intercept of each function as the rate of change, comparing -2 to -3 , and considers -3 to be greater since it is farther from 0 (i.e., compares the absolute values of the $x$-intercepts)
13. The table below represents a function of $x$.

| $x$ | $y$ |
| :---: | :---: |
| -5 | 0 |
| -3 | 2 |
| -1 | 4 |
| 1 | 6 |
| 3 | 8 |

Which statement about the function is true?
A. The function is increasing, has a $y$-intercept of 5 , and has a slope of 1 .
B. The function is decreasing, has a $y$-intercept of 5 , and has a slope of 1 .
C. The function is increasing, has a $y$-intercept of -5 , and has a slope of -1 .
D. The function is decreasing, has a $y$-intercept of -5 , and has a slope of ${ }^{-1}$.

Item Information

| Alignment | B-F.2.1.1 |
| :---: | :---: |
| Answer Key | A |
| Depth of Knowledge | 2 |
| $p$-value A | 51\% (correct answer) |
| $p$-value B | 10\% |
| $p$-value C | 27\% |
| $p$-value D | 12\% |
| Option Annotations | A. Correct: determines that the function is increasing since the $y$-values increase as the $x$-values increase, calculates the slope using the change in $y$-coordinates divided by the change in $x$-coordinates, resulting in $\frac{8-6}{3-1}=\frac{2}{2}=1$, and then uses the slope to determine that the line would pass through the point $(0,5)$, making the $y$-intercept 5 <br> B. sees the negative $x$-values and thinks this means the function is decreasing <br> C. uses the $x$-intercept as the $y$-intercept and switches either the $x$-values or the $y$-values when determining the slope <br> D. sees the negative $x$-values and thinks this means the function is decreasing, uses the $x$-intercept as the $y$-intercept, and switches either the $x$-values or the $y$-values when determining the slope |

14. The graph below models the number of people $(y)$ in a library based on the number of hours $(x)$ since the library opened.


Between which two consecutive hours since the library opened does the number of people in the library increase the most?
A. between 3 and 4 hours
B. between 4 and 5 hours
C. between 6 and 7 hours
D. between 7 and 8 hours

Item Information

| Alignment | B-F.2.1.2 |
| :--- | :--- |
| Answer Key | C |
| Depth of Knowledge | 2 |
| $p$-value A | $5 \%$ |
| $p$-value B | $6 \%$ |
| $p$-value C | $68 \%$ (correct answer) |
| $p$-value D | $21 \%$ |
| Option Annotations | A. selects the interval with the only increase (+6) after a decrease (-2) <br> B. selects an interval with no change <br> C. Correct: recognizes that from the 6-hour mark to the 7-hour mark, <br> the $y$-values increase from 16 to 26, which is the greatest increase <br> on the graph |
|  | D. selects a decreasing interval |

15. The scatter plot shown below represents the price and number of pages for each of 9 paperback books.


Which statement about the prices and numbers of pages for the paperback books represented in the scatter plot is true?
A. The point $(125,6)$ is an outlier.
B. There is a cluster of data at $x=400$.
C. The data show a positive correlation.
D. The data show a nonlinear association.

Item Information

| Alignment | D-S.1.1.1 |
| :--- | :--- |
| Answer Key | C |
| Depth of Knowledge | 1 |
| $p$-value A | $9 \%$ |
| $p$-value B | $18 \%$ |
| $p$-value C | $60 \%$ (correct answer) |
| $p$-value D | $13 \%$ |
| Option Annotations | A. considers the least or leftmost point to be an outlier <br> B. $\quad$defines a cluster as a set of data points for which more than 1 data <br> point has the same $x$-coordinate <br>  <br>  <br> C.Correct: recognizes that, in general, the $y$-values increase as the <br> $x$-values increase, and identifies this type of pattern as a positive <br> correlation <br> D.considers the association to be nonlinear since the data points do <br> not lie on a single line$\quad$ |

## OPEN-ENDED QUESTION

16. Some figures are plotted on the coordinate grid shown below.

A. Rounded to the nearest hundredth, what is the length, in units, of line segment PQ?
17. Continued. Please refer to the previous page for task explanation.

Figure $A$ is mapped onto figure $A^{\prime}$ without a rotational transformation.
B. Identify a set of two transformations that could map figure $A$ onto figure $A^{\prime}$ without using a rotational transformation.
transformation 1: $\qquad$
transformation 2: $\qquad$
C. Explain why figure $A$ is congruent to figure $A^{\prime}$.

A set of transformations maps figure $A$ onto figure $B$.
D. Explain why one of the transformations must be a dilation.

After you have finished your work, close this booklet so your teacher will know you are finished.

## Item-Specific Scoring Guideline

## \#16 Item Information

| Alignment | C-G.1 <br> C-G.2.1.3 | Depth of <br> Knowledge | 2 | Mean Score | 1.38 |
| :---: | :---: | :---: | :---: | :---: | :---: |

## Assessment Anchor this item will be reported under:

M08.C-G.1 - Demonstrate an understanding of geometric transformations.

## Specific Anchor Descriptor addressed by this item:

M08.C-G.1.1-Apply properties of geometric transformations to verify congruence or similarity.

## Scoring Guide

| Score | In this item, the student ... |
| :---: | :--- |
| $\mathbf{4}$ | Demonstrates a thorough understanding of geometric transformations by correctly <br> solving problems and clearly explaining procedures. |
| $\mathbf{3}$ | Demonstrates a general understanding of geometric transformations by correctly solving <br> problems and clearly explaining procedures with only minor errors or omissions. |
| $\mathbf{2}$ | Demonstrates a partial understanding of geometric transformations by correctly <br> performing a significant portion of the required task. |
| $\mathbf{1}$ | Demonstrates minimal understanding of geometric transformations. |
| $\mathbf{0}$ | The response has no correct answer and insufficient evidence to demonstrate any <br> understanding of the mathematical concepts and procedures as required by the task. <br> Response may show only information copied from the question. |

## Top-Scoring Student Response and Training Notes

| Score | Description |
| :---: | :--- |
| $\mathbf{4}$ | Student earns 4 points. |
| $\mathbf{3}$ | Student earns 3.0-3.5 points. |
| $\mathbf{2}$ | Student earns 2.0-2.5 points. |
| $\mathbf{1}$ | OR <br>  <br> $\mathbf{0}$ | | Student earns 0.5-1.5 points. |
| :--- |
| Response is incorrect or contains some correct work that is irrelevant to the skill or |
| conceng measured. |

## Top-Scoring Response

## Part A (1 point):

1 point for correct answer

OR $\frac{1}{2}$ point for using $\sqrt{10}$ or for rounding incorrectly

| What? |  |
| :---: | :--- |
| 3.16 (units) |  |
| OR equivalent |  |

## Part B (1 point):

1 point for correct answer

| What? | Why? |
| :--- | :---: |
| Answers may vary. Responses may be either two reflections (order does not <br> matter) such that the lines of reflection are perpendicular and intersect at the <br> origin or two dilations (order does not matter) such that the product of the <br> scale factors is -1 [Note: For the PSSA, students may use the informal term <br> "flip" rather than "reflection"; also, the center of dilation is the origin (unless <br> stated otherwise), so students are not required to include the center of dilation <br> as part of their response.]. |  |
| Sample Responses: |  |
| transformation 1: reflection across the $x$-axis |  |
| transformation 2: reflection across the $y$-axis |  |
| OR |  |
| transformation 1: dilation of -2 |  |
| transformation 2: dilation of 0.5 |  |

## Part C (1 point):

1 point for correct and complete explanation
OR $\frac{1}{2}$ point for correct but incomplete explanation

| What? | Why? |
| :--- | :--- |
|  | Sample Explanations: <br> Figure A is congruent to figure $A^{\prime}$ because the corresponding sides have equal <br> lengths and the corresponding angles have equal measurement. <br> OR <br> Figure A is congruent to figure A' because congruency is preserved under <br> reflections. <br> OR equivalent |

## Part D (1 point):

1 point for correct and complete explanation
OR $\frac{1}{2}$ point for correct but incomplete explanation

| What? | Why? |
| :--- | :--- |
|  | Sample Explanation: <br> Figure B is a larger version of figure A, with corresponding sides being in <br> proportion and corresponding angles being equal. The only transformation that <br> can increase/decrease the size of a figure, while preserving angle measures <br> and keeping side lengths proportional, is a dilation. |
| OR equivalent |  |

## STUDENT RESPONSE

## Response Score: 4 points



## PARTS A and B




Part A. The student provided the correct answer $(P Q=3.16)$. While support is not required for Part A , the student likely used the Pythagorean theorem to solve the equation $3^{2}+1^{2}=c^{2}$, simplified the left side as $9+1=10$, calculated $\sqrt{10}$ as approximately 3.162 , and then rounded to the nearest hundredth. [1 point]

Part B. The student correctly provided a set of two transformations (transformation 1: flip over the x-axis and transformation 2: flip over the $y$-axis) that could map figure A onto figure A' without using a rotational transformation. [1 point]

## PARTS C and D



Some figures are plotted on the coordinate grid shown below.
C. Explain why figure A is congruent to figure $\mathrm{A}^{\prime}$.

Figure A is congruent to figure $\mathrm{A}^{\prime}$ because the only transformations that occured to figure $A$ were flips over the $x$ and $y$ axis. Figure A did not get dialated or anything else, so it stayed the same size. Also, if you measure the sides, they are the same lengths.

261 / 1000

A set of transformations maps figure $A$ onto figure $B$.
D. Explain why one of the transformations must be a dilation.

One of the transformatons that must have happened to $A$ for it to become B is a dilation. You can clearly tell that figure B is bigger than figure A. If you can't, you can measure the lengths of the sides and you will see that figure $B$ is bigger.

245 / 1000


Part C. The student provided a correct explanation (Figure A did not get dialated . . . so it stayed the same size. Also, if you measure the sides, they are the same lengths) as to why figure $A$ is congruent to figure $A^{\prime}$. [1 point]

Part D. The student provided a correct explanation (figure $B$ is bigger than figure $A$ ) as to why one of the transformations must be a dilation to map figure $A$ onto figure $B$. [1 point]

## STUDENT RESPONSE

## Response Score: 3 points

16. Some figures are plotted on the coordinate grid shown below.

A. Rounded to the nearest hundredth, what is the length, in units, of line segment PQ?

$$
\text { The length of line segment } P Q \text { is }
$$



$$
3.25 \text { units long }
$$

Part A. The student provided an incorrect answer ( $\approx 3.25$ units). This answer is not correctly rounded or truncated. No support (work or explanation) is required, so it is unclear where an error was made. [0 points]
16. Continued. Please refer to the previous page for task explanation.

Figure $A$ is mapped onto figure $A^{\prime}$ without a rotational transformation.
B. Identify a set of two transformations that could map figure $A$ onto figure $A^{\prime}$ without using a rotational transformation.
transformation 1 : $\qquad$ A reflection across the $x$ axis transformation 2: $\qquad$ Areflection across the Maxis
Part B. The student correctly provided a set of two transformations (transformation 1: A reflection across the $x$ axis and transformation 2: A reflection across the $y$ axis). [1 point]
C. Explain why figure $A$ is congruent to figure $A^{\prime}$.
figure $A$ is Congruent to figure $A^{\prime}$ because
they are both the same shape and size.

Part C. The student provided a correct explanation (they are both the same shape and size) as to why figure $A$ is congruent to figure $A^{\prime}$. [1 point]

A set of transformations maps figure $A$ onto figure $B$.
D. Explain why one of the transformations must be a dilation.
one of the transformations must be a dilation
because the sige of the figure $A$ is half the size of figure $B$.

Part D. The student provided a correct explanation (they are both the same shape and size) as to why figure $A$ is congruent to figure $A^{\prime}$. [1 point]

After you have finished your work, close this booklet so your teacher will know you are finished.

## STUDENT RESPONSE

Response Score: 2 points


## PARTS A and B



Part A. The student provided an answer correctly rounded to the nearest tenth (3.2 units) rather than to the nearest hundredth. While support is not required for Part A, the student likely used the Pythagorean theorem to solve the equation $3^{2}+1^{2}=c^{2}$, simplified the left side as $9+1=10$, calculated $\sqrt{10}$ as approximately 3.162 , and then rounded to the nearest tenth. [ 0.5 points]

Part B. The student provided an incorrect set of two transformations/transformation 1: A reflection over x-axis and transformation 2: A dilation of $\frac{1}{2}$ ) that could map figure A onto figure A' without using a rotational transformation. Both transformations must be correct for credit to be awarded. [0 points]

## PARTS C and D



Part C. The student provided a correct but incomplete explanation (both figures have the same angle measurements) as to why figure $A$ is congruent to figure $A^{\prime}$. Figures with the same angle measures are similar but not necessarily congruent. A second comparison, such as observing that the two figures have the same size, same shape, or same side lengths, is needed for a complete explanation. [ 0.5 points]

Part D. The student provided a correct explanation (Figure $a$ is one size and figure $B$ is another) as to why one of the transformations must be a dilation to map figure A onto figure $B$. [1 point]

## STUDENT RESPONSE

## Response Score: 1 point

16. Some figures are plotted on the coordinate grid shown below.

A. Rounded to the nearest hundredth, what is the length, in units, of line segment $P Q$ ?



Part A. The student provided incorrect answers ( $2.1^{2}$ or $2^{2}$ ) for the length of line segment PQ. No support (work or explanation) is required, so it is unclear where an error was made. [ 0 points]
16. Continued. Please refer to the previous page for task explanation.

Figure $A$ is mapped onto figure $A^{\prime}$ without a rotational transformation.
B. Identify a set of two transformations that could map figure $A$ onto figure $A^{\prime}$ without using a rotational transformation.
transformation 1 :
 transformation
transformation 2:


Part B. The student provided an incorrect set of two transformations (transformation 1: Reflection transformation and transformation 2: Translation transformation) that could map figure A onto figure A' without using a rotational transformation. The first transformation does not specify what the figure is reflected over, and the second transformation is incorrect. Both transformations must be correct for credit to be awarded. [0 points]
 axis.

Part C. The student provided an incorrect explanation (they still run threw the 0.0 axis) as to why figure A is congruent to figure $\mathrm{A}^{\prime}$. [0 points]

A set of transformations maps figure $A$ onto figure $B$.
D. Explain why one of the transformations must be a dilation.

be a dilation because it has became bigger.

Part D. The student provided a correct explanation (it has became bigger) as to why one of the transformations must be a dilation to map figure A onto figure B . [1 point]

## STUDENT RESPONSE

## Response Score: 0 points



## PARTS A and B



Some figures are plotted on the coordinate grid shown below.

A. Rounded to the nearest hundredth, what is the length, in units, of line segment PQ?


Figure $A$ is mapped onto figure $A^{\prime}$ without a rotational transformation.
B. Identify a set of two transformations that could map figure A onto figure $\mathrm{A}^{\prime}$ without using a rotational transformation.
transformation 1: Both start at origin $(0,0)$
$\qquad$
$\square$
transformation 2: figure A is transferred across y axis

Part A. The student provided an incorrect answer (4) for the length of line segment PQ. No support (work or explanation) is required, so it is unclear where an error was made. The student may have added the two sides of the triangle $(3+1=4)$ to find the length of line segment PQ. [0 points]

Part B. The student provided an incorrect set of two transformations (transformation 1 : Both start at origin ( 0,0 ) and transformation 2: figure $A$ is transferred across $y$ axis) that could map figure A onto figure A' without using a rotational transformation. [0 points]

## PARTS C and D



Some figures are plotted on the coordinate grid shown below.

C. Explain why figure A is congruent to figure $\mathrm{A}^{\prime}$.

They are both mapped together.
$30 / 1000$

A set of transformations maps figure $A$ onto figure $B$.
D. Explain why one of the transformations must be a dilation.


Because it would not go across the $y$ axis.
$42 / 1000$

Part C. The student provided an incorrect explanation (They are both mapped together) as to why figure A is congruent to figure A'. [0 points]

Part D. The student provided an incorrect explanation (it would not go across the $y$ axis) as to why one of the transformations must be a dilation to map figure A onto figure $B$. [0 points]

## MATHEMATICS—SUMMARY DATA

## Multiple-Choice

| Sample <br> Number | Alignment | Answer Key | Depth of <br> Knowledge | $\boldsymbol{p}$-value <br> A | p-value <br> B | p-value <br> $\mathbf{C}$ | $\boldsymbol{p}$-value <br> $\mathbf{D}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | B-E.1.1.2 | A | 1 | $55 \%$ | $15 \%$ | $18 \%$ | $12 \%$ |
| 2 | B-E.1.1.3 | C | 2 | $18 \%$ | $14 \%$ | $53 \%$ | $15 \%$ |
| 3 | A-N.1.1.1 | B | 2 | $18 \%$ | $32 \%$ | $28 \%$ | $22 \%$ |
| 4 | A-N.1.1.5 | A | 2 | $39 \%$ | $34 \%$ | $14 \%$ | $13 \%$ |
| 5 | B-E.1.1.1 | A | D | 1 | $18 \%$ | $15 \%$ | $21 \%$ |
| 6 | B-E.1.1.4 | A | 2 | $34 \%$ | $30 \%$ | $17 \%$ | $19 \%$ |
| 7 | B-E.2.1 | C | 2 | $15 \%$ | $18 \%$ | $59 \%$ | $8 \%$ |
| 8 | B-E.2.1.1 | D | 2 | $13 \%$ | $7 \%$ | $9 \%$ | $71 \%$ |
| 9 | B-E.2.1.2 | D | 2 | $12 \%$ | $21 \%$ | $19 \%$ | $48 \%$ |
| 10 | B-E.2.1.3 | C | 1 | $24 \%$ | $14 \%$ | $52 \%$ | $10 \%$ |
| 11 | B-E.3.1.1 | C | 1 | $14 \%$ | $16 \%$ | $56 \%$ | $14 \%$ |
| 12 | B-F.1.1.2 | A | 2 | $45 \%$ | $18 \%$ | $25 \%$ | $12 \%$ |
| 13 | B-F.2.1.1 | A | 2 | $51 \%$ | $10 \%$ | $27 \%$ | $12 \%$ |
| 14 | B-F.2.1.2 | C | 2 | $5 \%$ | $6 \%$ | $68 \%$ | $21 \%$ |
| 15 | D-S.1.1.1 | C | 1 | $9 \%$ | $18 \%$ | $60 \%$ | $13 \%$ |

## Open-Ended

| Sample <br> Number | Alignment | Points | Depth of <br> Knowledge | Mean Score |
| :---: | :---: | :---: | :---: | :---: |
| 16 | C-G.1 <br> C-G.2.1.3 | 4 | 2 | 1.38 |

## PSSA Grade 8 Mathematics Item and Scoring Sampler

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[^0]:    1 The permission to copy and/or use these materials does not extend to commercial purposes.

[^1]:    2 All $p$-value percentages listed in the item information tables have been rounded.

