Write functions or sequences that model relationships between Calculate the slope of a pedestrian ramp $=$ two quantities

Program Task: Calculate the slope of a pedestrian ramp.

## Program Associated Vocabulary: <br> SLOPE, RISE, RUN, AMERICAN WITH DISABILITIES ACT (ADA)

## Program Formulas and Procedures:

The Americans with Disabilities Act (ADA) was established in 1990. It continues to evolve and become more stringent. The ADA is a federal law, not a building code. The law states that a pedestrian ramp cannot exceed a slope of $1 / 12$, that is, for every 12 units (inches, feet, etc.) of horizontal distance, it cannot rise more than 1 unit (inches, feet, etc.). There are other regulations pertaining to ramps, but we will limit this sample to finding a slope.

Example: You work for an architect and are asked to design a ramp connecting two sidewalks. You have to make sure the ramp does not exceed the $1: 12$ slope mandated by the ADA. Calculate the slope if the distance is $20^{\prime}$, the lower sidewalk is at an elevation of 100.00 ' feet and the upper sidewalk is at an elevation of $101.25^{\prime}$. Will it comply?

Note that elevations on drawings are normally done by civil engineers using decimals rather than feet and inches. These elevations reference the height of the ground above sea level.


Subtract the higher elevation from the lower elevation:
Sidewalk $1=101.25$ '
Sidewalk $2=\frac{100.00^{\prime}}{1.25}$, $1.25^{\prime}$ (change in elevation)

Convert the above to feet and inches: ( 1 ' -3 ")
Since the law states that you cannot exceed a slope of $1: 12$, you need to determine the existing slope.

Since you may have 1 unit of rise for every 12 units of run ( 1 " per foot), a 20 ' ramp has a maximum rise of $20 "$ " 1 " x 20 units $=20$ ").

20 inches $=1^{\prime}-8^{\prime \prime}$
Since the $1^{\prime}-3$ " you determined above is less than the $1^{\prime}-8$ " allowed, you can design a legal ramp for this location.

## Pa Core Standard: CC.2.2.HS.C. 3

Description: Write functions or sequences that model relationships between two quantities.

## Math Associated Vocabulary:

SLOPE, RISE, RUN, RATE OF CHANGE, LINE, $\Delta \mathrm{X}, \Delta \mathrm{Y}$

## Formulas and Procedures:

slope $=\frac{Y_{2}-Y_{1}}{X_{2}-X_{1}}=\frac{\text { Rise }}{\text { Run }}=\frac{\Delta Y}{\Delta X}$


To find the slope of the line above:
Step1: Label your coordinates $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$. $*$ Note: It does not matter which coordinate you select to represent $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and ( $\mathrm{x}_{2}$, $\mathrm{y}_{2}$ ).

Step 2: Substitute values into the formula and solve. For our example, we'll make $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=(1,2)$ and $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)=(5,3)$.

$$
\text { slope }=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{3-2}{5-1}=\frac{1}{4}
$$

*Note: Slope is written as a fraction in simplest form.

## Drafting \& Design Technology/Technician (15.1301) T-Chart

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## Instructor's Script - Comparing and Contrasting

The formula for slope taught in a math classroom uses Cartesian coordinate points to calculate the slope of a line. The change in yvalues (rise) is divided by the change in $x$-values (run). The example displayed for drafting uses a blueprint sketch with measurement values as opposed to coordinates, but the principle is similar. In both scenarios, rise over run must be calculated. The drafter must subtract the two elevations to find the rise and the run given as the length of the ramp. In a Math class, the slope can be either positive or negative, but in the real world, slopes are expressed as positive numbers. The drafting example is more complex than what is required of the eligible content because not only must students find the slope of the ramp, they must compare it to the slope value required to be in ADA compliance. The value of the slope of the ramp is $1.25 / 20$. The student must be able to compare that value to $1 / 12$ and evaluate if the actual slope is greater than or less than $1 / 12$. This requires the use of additional PA Common Core Standards:
CC.2.1.HS.F. 2

Apply properties of rational and irrational numbers to solve real world or mathematical problems. Expressing the two slopes in decimal form allow us to compare the two values. Remember, the larger the slope, the steeper it is.
$1.25 / 20=0.06251 / 12=0.0833$, Since $0.0625<0.0833$, the ramp is in compliance.
CC.2.1.7.D. 1

Analyze proportional relationships and use them to model and solve real-world and mathematical problems. It's difficult to compare fractions with different denominators. We can set up a proportion to convert $1.25 / 20$ to a fraction with 12 as the denominator.
$\frac{1.25}{20}=\frac{x}{12} \rightarrow 1.25(12)=20 x \rightarrow 15=20 x \rightarrow \frac{15}{20}=\frac{20 x}{20} \rightarrow 0.75=x$, Therefore the slope is $0.75 / 12$.

## Common Mistakes Made By Students

Often students forget that the $y$-coordinates go in the numerator and the $x$-coordinates go in the denominator.
Students will often not subtract consistently among y and $x$ values. For instance, for the slope of a line passing through the points $(3,5)$ and $(-1,7)$ :

$\frac{7-5}{3-(-1)}$ or $\frac{5-7}{-1-3}$
INCORRECT
instead of the correct answer:


## CTE Instructor's Extended Discussion

The ADA publishes the Americans with Disabilities Act Accessibility Guidelines (ADAAG). While we have only touched on one section of this act (ramps), it is critical that architectural drafting students are aware of and know how to use the guidelines. They address such things as running and cross slope of sidewalks (running slope max. $1 / 20$, cross slope max. $1 / 48$ ), ramps, widths of doors, and heights of countertops, urinals, toilet paper dispensers, etc. Being aware of these requirements and knowing how to use them will prevent costly errors.

Drafters should not rely on the architect being right every time they are handed a sketch and told to draft it. Very often the architect or designer will make a math error, and if it is not caught by the drafter, the ramp, sidewalk, etc. may be built incorrectly, will fail inspection, and have to be torn out and replaced. You can be sure that the owner will ask the architect to pay for it since the architect was the one who made the error in the first place.


| Problems Career and Tec | ical Math Concepts Solutions |
| :---: | :---: |
| 1. Using the sample problem determine if a ramp would be legal with the following information: <br> Sidewalk Elevations: $102.25^{\prime}$ and $103.75^{\prime}$, Distance $=14^{\prime}$ | $103.75^{\prime}-102.25^{\prime}=1.50^{\prime}:\left(1^{\prime}-6^{\prime \prime} \text { rise }\right)$ <br> 1 " of rise x 14 units of run = 14 ": ( $1^{\prime}-2$ " rise $)$ <br> Since the rise you calculated exceeds the rise allowed, a straight ramp would not be legal. You would have to design a ramp with a switchback to comply with the law. |
| 2. Using the sample problem determine if a ramp would be legal with the following information: <br> Sidewalk Elevations: 300.00' and 301.42', Distance $=18^{\prime}$ | $301.42^{\prime}-300.00^{\prime}=1.42^{\prime}:\left(1^{\prime}-5^{\prime \prime}\right.$ rise $)$ 1 " of rise x 18 units of run = 18 ": ( 1 '- 6 " rise ) Since the rise you calculated is 1 " less than the allowed slope, you can design a legal ramp. |
| 3. Using the sample problem determine if a ramp would be legal with the following information: <br> Sidewalk Elevations: 200.00' and 201.25', Distance $=20^{\prime}$ | $201.25^{\prime}-200.00^{\prime}=1.25^{\prime}:\left(1^{\prime}-3^{\prime \prime}\right.$ rise $)$ <br> 1 " of rise x 20 units of run = $20 "$ : ( $1^{\prime}-8$ " rise $)$ <br> Since the rise you calculated is $5 "$ less than the allowed slope, you can design a legal ramp. |
| Problems Related, Gene | Math Concepts Solutions |
| 4. A ramp increases from ground level to a height of 5 feet over a span of 20 feet. What is the slope (rate of change) of the ramp? | $\frac{5}{20}=\frac{1}{4}$ |
| 5. Determine the slope of the line graphed at the right: |  $\begin{aligned} & \mathrm{m}=\frac{4-2}{4-0} \\ & =\frac{2}{4}=\frac{1}{2} \end{aligned}$ |
| 6. A sidewalk increases from ground level to a height of 3 feet over a span of 40 feet. What is the slope (rate of change) of the sidewalk? | $\frac{3}{40}$ |
| Problems PA Core | Math Look Solutions |
| 7. Find the slope of a line passing through the points $(3,5)$ and $(2,1)$. | $\frac{5-1}{3-2}=\frac{4}{1}=4 \quad \text { or } \quad \frac{1-5}{2-3}=\frac{-4}{-1}=4$ |
| 8. Find the slope of a line passing through the points $(-2,1)$ and (4, -5). | $\frac{-5-1}{4-(-2)}=\frac{-6}{6}=-1 \quad \text { or } \quad \frac{1-(-5)}{-2-4}=\frac{6}{-6}=-1$ |
| 9. Find the slope of a line passing through the points $(4,2)$ and $(-5,6)$. | $\frac{6-2}{-5-4}=\frac{4}{-9}=-\frac{4}{9} \quad \text { or } \frac{2-6}{4-(-5)}=\frac{-4}{9}=-\frac{4}{9}$ |

