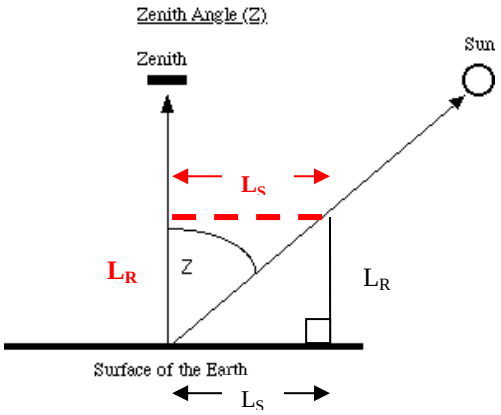
Calculate the sun's zenith angle = **Apply trigonometric ratios to solve problems involving right triangles**

Program Task: Describe the characteristics of solar irradiance on Earth

Program Associated Vocabulary:
ZENITH, SOLAR RADIATION, AIR MASS

Program Formulas and Procedures:
This task relates to the use of solar energy in building construction and energy use. Finding the zenith angle will allow builders to accurately attach solar panels to achieve maximum efficiency and also allow air mass calculations. The sun's zenith angle for air mass calculations can be determined by using a vertical stake or ruler of known height and measuring the length of the shadow cast. The ruler, shadow, and rays of the sun form a triangle. Using trigonometry, the zenith angle can be calculated with the following formula:



$$Z = \Theta_Z = \tan^{-1} \frac{L_S}{L_R}$$

Where
Z = zenith angle (= Θ_z)
L_S = length of shadow in inches
L_R = length of ruler in inches
**Opposite sides of a rectangle are equal.

Example: A student is trying to find the zenith angle of the sun. He puts a ruler into the earth at a 90^0 angle. If the length of a ruler is 12" and the length of the sun's shadow is 3", then what is the zenith angle of the sun?

Step 1: Complete the rectangle and see that opposite sides of a rectangle are equal.
Step 2: Choose trig function $Tan^{-1}(L_S/L_R)$
Step 3: Substitute the values and solve.
$Tan^{-1}(3/12) \approx 14^0$

PA Core Standard: CC.2.3.HS.A.7

Description: Apply trigonometric ratios to solve problems involving right triangles

Math Associated Vocabulary:
ANGLE, RIGHT TRIANGLE, TRIGONOMETRIC RATIOS, INVERSE TRIGONOMETRIC RATIOS, RECTANGLE RULES

Formulas and Procedures:
- A right angle is 90 degrees.
- A right triangle has one right angle.
- Ask students to find right triangles within the classroom or to think of other real-life examples.
- Legs are sides next to right angle; the hypotenuse is the side opposite the right angle and is also the longest side.
- Draw a right triangle on the board. Have students draw it in their notes. Have them identify the legs and the hypotenuse in their notes. Show the answer on the board so they can check their work.
- Opposite sides of a rectangle are equal.

SOH CAH TOA
Sine of the angle = length of opposite leg divided by length of hypotenuse

Cosine of the angle = length of adjacent leg divided by length of hypotenuse

Tangent of the angle = length of opposite leg divided by length of adjacent leg

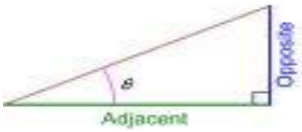
$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

SOH

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

CAH

TOA



$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

Example: Find the angle of a right triangle with an adjacent side of 15, and an opposite side of 25.

Step 1: Determine the operation. Tan = Opposite/Adjacent
Step 2: Since you are looking for the angle, it is an inverse.
Step 3: Substitute values and solve: $Tan^{-1}(25/15) \approx 59^0$

Instructor's Script – Inverse Trigonometric Ratios
In mathematics, every operation has an opposite operation that undoes it. Examples: adding and subtracting, multiplying and dividing, squaring and square rooting. These are all known as inverses.

Sometimes the angle is the unknown value and we need to be able to take a ratio and find the angle (work backwards). To do this, we need to use inverse trig functions, written as sin, cos, and tan to the -1 power.

Make sure students know how to find the inverse sin, cos, and tan using their calculators. Have them practice several problems. Also, students need to know that rectangles have opposite sides that are equal in length.

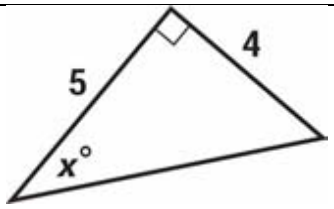
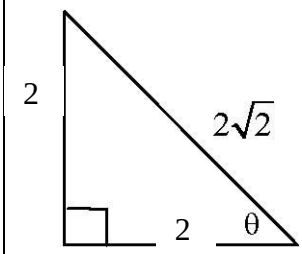
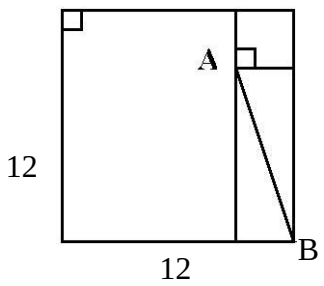
Common Mistakes Made By Students
Calculator settings

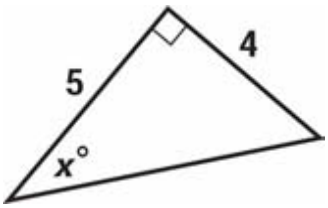
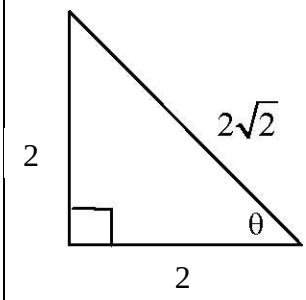
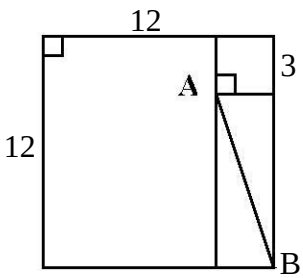
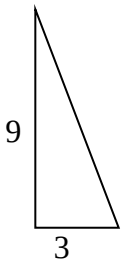
- Make sure that the mode of the calculator is set to the correct unit for the angle measurements being used (probably degree mode rather than radian mode).
- Enter trig functions in some (most graphing calculators) in the order written.
- In some calculators, the angle is entered before the trig function.
- When calculating angles, generally use the 2nd or shift key with the trig functions to obtain the inverse trig functions.
- Since calculators follow the order of operations, when entering complex fractions into the calculator, use parenthesis around the numerator and the denominator, or calculate each separately before dividing.

Trig Errors

- Make sure the correct formula is chosen. Discuss which angle you are looking for and what the different sides mean.
- Make sure rounding of angles is allowed or a discussion on converting decimals to minutes and seconds is necessary.

CTE Instructor's Extended Discussion
Air mass is a representation of the relative thickness of atmosphere that solar radiation must pass through to reach a point on the Earth's surface. When the sun is at its **zenith,** the amount of atmosphere that the sun's rays have to pass through to reach the Earth's surface is at a minimum. Zenith is the point in the sky directly overhead a particular location. The **zenith angle** is the angle between the sun and the zenith. As the zenith angle increases, the sun approaches the horizon and the sun's rays must pass through a greater amount of atmosphere to reach the Earth's surface. The air mass formula is less accurate for zenith angles greater than about 60 degrees.

Problems	Occupational (Contextual) Math Concepts	Solutions
1. Calculate the zenith angle if the length of the shadow for a 12 inch ruler is 16 inches.		
2. Calculate the zenith angle if the length of the shadow for a yardstick is 30 inches.		
3. Calculate the zenith angle if the length of the shadow for a 15 inch ruler is 16 inches.		
Problems	**Related, Generic Math Concepts**	**Solutions**
4. $\tan^{-1} .53 =$ _____		
5. $\tan x = \frac{7}{8}$, x = _____		
6.  x = _____		
Problems	**PA Core Math Look**	**Solutions**
7. Consider the following right triangle. Which of the following statements is correct?  a) $\sin\theta < \cos\theta$ b) $\sin\theta > \cos\theta$ c) $\sin\theta = \cos\theta$ d) $\sin\theta = 2$		
8. Using the drawing, including the 2 squares, which inequality gives the **best approximation** to AB?  a) $9 < AB < 10$ b) $11 < AB < 12$ c) $10 < AB < 11$ d) $9 < AB < 12$		

Problems	Occupational (Contextual) Math Concepts	Solutions
1. Calculate the zenith angle if the length of the shadow for a 12 inch ruler is 16 inches.		$Z = \theta_Z = \tan^{-1}\frac{16}{12} \approx 53°$
2. Calculate the zenith angle if the length of the shadow for a yardstick is 30 inches.		1 yardstick = 36” $Z = \theta_Z = \tan^{-1}\frac{30}{36} \approx 40°$
3. Calculate the zenith angle if the length of the shadow for a 15 inch ruler is 16 inches.		$Z = \theta_Z = \tan^{-1}\frac{16}{15} \approx 47°$
Problems	**Related, Generic Math Concepts**	**Solutions**
4. $\tan^{-1}$ .53 = _____ (Rounded to the nearest tenth.)		27.9° (In TI-30 Calculator**:** 2nd Key→Tan →.53 → =)
5. tan x = ⅞, x = _____(Rounded to the nearest tenth)		41.2° (In TI-30 Calculator: 2nd Key→Tan→ .7/8 → =)
6.  x = ____ Round to tenth		38.7° Determine Trig Formula: Inv TAN = Opp/Adj $\text{Tan}^{-1}(4/5) = 38.7^0$
Problems	**PA Core Math Look**	**Solutions**
7. Consider the following right triangle. Which of the following statements is correct?  a) sin θ < cos θ b) sin θ > cos θ c) sin θ = cos θ d) sin θ = 2		Set up formulas and compare. Sin = Opp/Hyp = $2/2\sqrt{2}$ Cos = Adj/Hyp = $2/2\sqrt{2}$ The correct answer is c.
8. Using the drawing, including the 2 squares, which inequality gives the **best approximation** to AB?  a) 9 < AB < 10 b) 11 < AB < 12 c) 10 < AB < 11 d) 9 < AB < 12		**Step 1**: Determine that all sides of small square are 3 and all sides of the large sq. are 12. **Step 2**: Pull triangle out and label sides. The long sides of the triangle is 12 - 3 = 9 **Step 3**: $\text{Tan}^{-1}(9/3) \approx 71.6^0$ **Step 4**: Sin 71.6^o = 9/Hyp Hypotenuse = $9/\text{Sin } 71.6^0 \approx 9.5$ (A.)  The correct answer is c. The Pythagorean Theorem could also be used.