

The Framework for Mathematics Coach Endorsement Guidelines

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**COMMONWEALTH OF PENNSYLVANIA
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Definition of an Endorsement Certificate

Following approval by the Department, baccalaureate or graduate degree granting institutions, alone or in cooperation with other institutions, community colleges or school entities, may offer short programs (12 credits) that lead to a Program Endorsement. The Program Endorsement provides knowledge in new and emerging areas where formal certification does not exist. The Program Endorsement is intended to improve a teacher's skills in dealing with complex classroom settings. These endorsements would be added to existing level I or level II certificates but are not required to perform service in these areas.

Introduction to Mathematics Coach Endorsement Guidelines

This document describes the professional knowledge, skills, and competencies that candidates in the Mathematics Coach Endorsement program will learn by completing a prescribed sequence of courses (including field placements). In addition to specific requirements and competencies, these guidelines discuss the mathematics coach program design, candidate competencies, field experiences, and any prerequisite certificates needed by the candidate.

Prerequisite for Mathematics Coach Endorsement Program:

To be admitted into a Mathematics Coach Endorsement Program, a candidate must have at least three years of successful mathematics teaching experience and Instructional I or II certificate in one or more of the following areas: Elementary Education K-6; PreK-4; 4-8; or 7-12 Mathematics.

All endorsement programs must include some components of field experiences. (22 Pa. §354.25(d)). Field experiences are defined as a range of formal, required school and community activities participated in by candidates who are enrolled in educator preparation programs. These activities generally include supervision and mentorship of a teacher with expertise in the endorsement area. Effective field experiences provide candidates with increasing exposure to learning situations and school settings under the

guidance of program faculty and trained mentors, throughout the preparation program. The mathematics coach endorsement program requires a candidate to complete the field experience competencies.

Institutions must explain:

1. How they implement field experiences.
2. The duration of candidate field experiences.
3. How these experiences are closely integrated with coursework, assessment practices, and program goals.

At least one experience must include students in inclusive settings. An inclusive setting is defined as an educational setting which includes students with and without special needs. An inclusive setting includes at least one child with an Individual Family Service Plan/ Individualized Education Program. At least one experience must be in a public school setting.

Overview of Mathematics Coach Endorsement Program

Research is showing mathematics coaches have a positive impact in schools, even narrowing the achievement gap between certain categories of students (Meyers and Harris 2008), (Campbell and Malkus, 2009), (Brosan and Erchick, 2009) . Mathematics coaches are teacher leaders with a strong preparation and background in mathematics content, mathematics pedagogical content knowledge, instructional strategies, and school leadership. They are former classroom teachers who are responsible for supporting the professional growth of their colleagues and promoting enhanced mathematics instruction and student learning throughout their school. They are responsible for strengthening classroom teachers' understanding of mathematics content, and helping teachers develop more effective mathematics teaching practices that allow all students to reach high standards as well as sharing research addressing how students learn mathematics.

Mathematics Coach Endorsement Program Design

The professional core courses, competencies, and experiences for the Mathematics Coach Endorsement Program should be designed to address the specific set of mathematics content, pedagogical content knowledge, issues, and competencies that are relevant to teaching and learning mathematics. The program must prepare coaches who will be able to support teachers' efforts to help students master the Pennsylvania Core standards in mathematics and the mathematics behaviors and practices associated with being mathematically proficient. The Pennsylvania Department of Education believes that educator certification programs should be comprehensive and delivered through a combination of classroom, school, and other appropriate settings. While some online courses may be a component of the program, programs should include face-to-face components. Field experiences, for example, must be face-to-face.

The program should consist of required competencies and include on-site field experiences. The program design must describe clearly how the relevant set of knowledge, skills and competencies will be addressed. The application must also indicate how the institution will assess whether candidates have acquired the required knowledge, skills and competencies. All mathematics education courses and related education courses should be grounded in theories of cognitive, emotional and social development, and should enable candidates to gain the knowledge and experience to work successfully with all teachers of mathematics and the school community.

A mathematics coach (PreK–12) must:

1. Be an effective mathematics content teacher at the PreK-4, 4-8 or secondary level.
2. Hold an Instructional I or II certificate, and have a minimum of three years of effective teaching experience on the certificate.
3. Complete an approved Mathematics Coach Program.

Candidate Competencies

This section outlines the competencies required for the endorsement by Chapter 49 of the Pennsylvania School Code: "The preparing institution shall ensure that candidates complete a well-planned sequence of professional educator courses and field experiences to develop an understanding of the structure, skills, core concepts, facts,

methods of inquiry and application of technology related to each academic discipline the candidates plan to teach or in the academic disciplines related to the non-instructional certificate categories in which they plan to serve.” (22 Pa Code §354.25(a) (3)).

Aligned resources and tools to support the acquisition of these competencies can be found on the [Standards Aligned System \(SAS\) portal](#).

I. Content Knowledge for Teaching Mathematics

Candidates must know and understand deeply the mathematics across PK-12 as well as how mathematics concepts and skills develop throughout the grade levels. This knowledge includes specialized knowledge that teachers need to understand and support student learning of mathematics.

Candidates are expected to acquire the habits of mind of a mathematical thinker and use mathematical practices such as persevering in problem solving, precision in language, construction and comparison of mathematical representations, conjecturing, reasoning, and proving. They need to be able to use these practices in the following domains:

II. Number and Operations

Candidates will demonstrate their abilities in and understanding of:

Number and Operations

- Pre-number concepts: Non-quantified comparisons (less than, more than, the same), containment (e.g., 5 contains 3), subitizing, 1-to-1 correspondence, cardinality, ordinality.
- A comprehensive repertoire of interpretations of the four operations of arithmetic and of the common ways they can be applied.
- Place value: The structure of place-value notation in general and base-10 notations in particular, how place-value notations efficiently represent even very large numbers, as well as decimals and use of these notations to order numbers, estimate, and represent order of magnitude (e.g., using scientific notation).
- Multi-digit calculations including standard algorithms, mental math, and non-standard ways commonly created by students, and informal reasoning used in calculations.
- Basic number systems: Whole numbers (non-negative integers), integers, non-negative rational numbers, rational numbers, complex numbers, and real numbers. Relationships among them, and locations of numbers in each system on the number line. What is involved in extending operations

from each system (e.g., whole numbers) to larger systems (e.g., rational numbers).

- Multiplicative arithmetic: Factors, multiples, primes, least common multiple, greatest common factor. Proportional reasoning and rescaling.

III. Algebra and Functions

Candidates will demonstrate their abilities in and understanding of:

- Axioms: Recognize commutativity, associativity, and distributivity, and 0 and 1 as identity elements in the basic number systems. Understand how these may be used in computations and to deduce the correctness of algorithms. The need for order-of-operations conventions.
- Algebraic notation and equations: Literal symbols, as shorthand names for mathematical objects, or in the case of numerical variables as indicating an unspecified member of some class of numbers (the “range of variation”). The process of substitution of particular numbers into variable expressions. The solution set of an algebraic equation or relations.
- Transformations of equations (or relations) that do not change the solutions set.
- Modeling of problems, both mathematical and “real world,” using algebraic equations and relations.
- The concept of a function as defining one variable uniquely in terms of another. Familiarity with basic types of functions, including constant, linear, exponential, and quadratic. Representations and partial representations of functions: formula, graph, table or, when the variable is discrete, by recursion.
- Finding functions to model various kinds of growth, both numerical and geometric.

IV. Geometry and Measurement

Candidates will demonstrate their abilities in and understanding of:

- Visualization: Geometric objects are pictured on a 2-dimensional page, for 3-dimensional objects this requires perspective or projection renderings. Producing and reading such representations calls for special skills, both mathematical and drawing.
- Composing and decomposing: A geometric figure can be assembled by joining together various component figures. Conversely, a geometric figure may be decomposed into pieces, for example decomposing a polygon into an assemblage of triangles.

- Congruence and similarity: Congruence is the basic concept of geometric “sameness.” Similarity has to do with rescaling: Two figures are similar if one of them is congruent to a rescaling of the other. For example, all circles are similar, as are all squares and all isosceles right triangles.
- Geometric measurement is a way of attaching a numerical quantity to a geometric figure. Doing this involves a choice of some standard figure (the “unit”) and then the measurement is a kind of ratio of the given figure to the unit or, put differently, how many copies of the unit does it take to compose the given figure? It follows that if a geometric figure is decomposed, then its measure is the sum of the measures of its components. Changing the unit has the effect of multiplying all measurements by a constant (relating the two units). For example, relating feet to inches, or to meters.
- Common units of geometric measurement:
 - Linear: The unit may be the interval $[0, 1]$ on the number line.
 - Area: The unit is a unit square.
 - Volume: The unit is the unit cube.
 - Angle: Draw a unit circle centered at the vertex of the angle, and consider the arc of the circle cut out by the angle. The *radian* measure of the angle is the length α of that arc. The *degree* measure of the angle is $360\alpha/2\pi$ (i.e., 360 times the fraction of the circumference of the circle formed by the arc).
- Basic geometric figures in each dimension:
 - Dimension 1: Line segments and arcs of circles.
 - Dimension 2: Polygons and circles.
 - Dimension 3: Polyhedral solids, cylinders, cones and spheres.
 Elements of these figures (e.g., vertex, edge, face). Properties of regularity and symmetry. Definitions, names, and classification. Various kinds of measurement including surface area and volume; invariance under congruence, and behavior under rescaling.
- Plane coordinates: How they are introduced, and how they support algebraic expression of geometric objects and relationships. Reciprocally, how they afford geometric interpretation of algebraic relations.
- Transformations: Reflections, rotations, translations, dilations, symmetry and its expression in terms of transformation (e.g., reflection through a line of symmetry).
- Proof: Making and proving conjectures about geometric shapes or relations.

V. Data Analysis and Probability

Candidates will demonstrate their abilities in and understanding of:

- The nature and uses of data: What kinds of questions require data for their answers, and what kinds of data are required? How are relevant data sets created and organized?
- Designing an investigation, including specification of how the data collected support analysis responsive to the question(s) under investigation.
- Distinguish categorical (discrete) data (e.g., gender, favorite ice cream flavor) from measurement (continuous) data.
- Appropriate types of representation of data, and what they afford: For categorical data and relative frequencies. For measurement data, displays of shape, center and spread.
- Basic concepts of probability and ways to represent them, making judgments under conditions of uncertainty, measuring likelihood and becoming familiar with the concept of randomness.
- Drawing conclusions: Understand which representations best support communication of inferences from data, use probability models when appropriate, and account for variability.
- Understand the limits of generalizability due to non-randomness of a sample population.

VI. Specialized Mathematics Knowledge for Teaching

Candidates will demonstrate their abilities in and understanding of:

- The development of mathematical proficiency as characterized by conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (National Research Council, 2001).
- Opportunities for learners to develop mathematical practices, and to critically evaluate their selection and use of these practices.
- Diagnosing mathematical misconceptions and errors, and design appropriate interventions.
- Deciding whether, how and how far, to utilize specific oral or written responses from learners.
- Recognizing, evaluating, and responding to multiple, often non-standard solutions to problems.
- Choosing and/or designing tasks to support the learning of new mathematical ideas or methods, or to test learners' understanding of them.

Professional Knowledge for Teaching

Candidates are expected to have a foundation in *pedagogical content knowledge* (PCK) (Ball, Thames, & Phelps, 2008).

A. Learners and Learning

Candidates will demonstrate their abilities in and understanding of:

- Utilizing and building upon learners' existing knowledge, skills, understandings, conceptions and misconceptions to advance learning.
- Understanding learning trajectories related to particular topics in mathematics (e.g., Sarama & Clements, 2009) and use this knowledge to organize and deliver instruction that is developmentally appropriate and responsive to individual learners.
- Understanding cultural differences among learners (e.g., algorithms or learning practices familiar to different groups of learners) and utilize this knowledge to motivate and extend learning opportunities for individuals and groups of learners.
- Creating social learning contexts that engage learners in discussions and mathematical explorations among peers to motivate and extend learning opportunities.

B. Teaching

Candidates will demonstrate their abilities in and understanding of:

- Designing, selecting and/or adapting worthwhile mathematics tasks and sequences of examples that support a particular learning goal.
- Supporting students' learning of appropriate technical language associated with mathematics, attending to both mathematical integrity and usability by learners.
- Constructing and evaluating multiple representations of mathematical ideas or processes, establish correspondences between representations, and understand the purpose and value of doing so.
- Using questions to effectively probe mathematical understanding and make productive use of responses.
- Developing learners' abilities to give clear and coherent public mathematical communications in a classroom setting.
- Modeling effective problem solving and mathematical practices—questioning, representing, communicating, conjecturing, making connections, reasoning and proving, self-monitoring and cultivate the development of such practices in learners.
- Using various instructional applications of technology, judiciously, in ways that are mathematically and pedagogically grounded.

- Analyzing and evaluating student ideas and work, and design appropriate responses.
- Developing skillful and flexible use of different instructional formats—whole group, small group, partner, and individual—in support of learning goals.
- Managing diversities of the classroom and school—cultural, disability, linguistic, gender, socio-economic, developmental—and use appropriate strategies to support mathematical learning of all students.

C. Curriculum and Assessment

Candidates will demonstrate their abilities in and understanding of:

- Learning trajectories related to mathematical topics and use this knowledge to sequence activities and design instructional tasks.
- Using multiple strategies, including listening to and understanding the ways students think about mathematics, to assess students' mathematical knowledge.
- Understanding the importance of careful sequencing and development of mathematical ideas, concepts, and skills in the PreK–middle grades curriculum and be able to engage in discussions and decision-making to establish appropriate benchmarks for learning goals through grade eight.
- Selecting, using, adapting, and determining the suitability of mathematics curricula and teaching materials (e.g., textbooks, technology and manipulatives) for particular learning goals.
- Evaluating the alignment of local and state curriculum standards, district textbooks, and district and state assessments, and recommend appropriate adjustments to address gaps.
- Knowing the different formats and purposes, uses, and limitations of various types of assessment of student learning; be able to choose, design, and/or adapt assessment tasks for monitoring student learning.
- Using diagnostic instruments to analyze and identify student learning difficulties and develop intervention plans.
- Using the formative assessment cycle (administer a formative assessment task, analyze student responses to the task, and design and reteach lessons based on this analysis) and be able to find or create appropriate resources for this purpose.
- Analyzing formative and summative assessment results, make appropriate interpretations and communicate results to appropriate and varied audiences.

D. Leadership Knowledge and Skills

Candidates will demonstrate their abilities in and understanding of:

- Using professional resources such as professional organization networks, journals, and discussion groups to be informed about critical issues related to

- mathematics teaching and learning, (e.g., policy initiatives and curriculum trends).
- Selecting from a repertoire of methods to communicate professionally about students, curriculum, instruction, and assessment to educational constituents—parents and other caregivers, school administrators, and school boards.
 - Planning, developing, implementing, and evaluating professional development programs at the school and district level and support teachers in systematically reflecting and learning from practice.
 - Evaluating educational structures and policies that affect students' equitable access to high quality mathematics instruction, and act professionally to assure that all students have appropriate opportunities to learn important mathematics.
 - Using leadership skills to improve mathematics programs at the school and district levels, e.g., develop appropriate classroom- or school-level learning environments; build relationships with teachers, administrators and the community; develop evidence-based interventions for high and low-achieving students; collaborate to create a shared vision and develop an action plan for school improvement; partner with school-based professionals to improve each student's achievement; mentor new and experienced teachers to better serve students.

Faculty

Endorsement programs submitted for review to the Pennsylvania Department of Education will include the qualifications of faculty assigned to teach each course within the program. Faculty who teach in the mathematics coach endorsement program must have expertise in mathematics, teaching experience in a PK-12 setting, and advanced degrees in mathematics education. They must also be active in professional organizations related to mathematics education. Additionally, program proposals will be expected to include evidence of collaboration with current practicing mathematics coaches and mathematics supervisors.

Programs may be approved if at least 80 percent of faculty members for this program are qualified to teach their assigned course(s). If any faculty are found to be unqualified for their assigned course(s), the institution will have two years to ensure that 100 percent of the faculty are qualified. Evidence of qualification includes related academic degrees, public school certification(s), professional experience in basic (PreK-12) and/or higher education, and professional development pertaining to the competencies assigned to a course.